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398. Proposed by C. N. SCHMALL, New York City.

In a square $ABCD$ draw the diagonal AC . Now bisect AD in G and draw GB cutting AC in H . Prove that $\triangle AGH = \frac{1}{2} \triangle CGH = \frac{1}{3} \triangle ABG = \frac{1}{4} \triangle BCH$.

Solution by A. H. HOLMES, Brunswick, Maine, and G. I. HOPKINS, A. M., Manchester, New Hampshire.

Since $BC=2AG$ by construction, and $\angle CBH = \angle GAH$, $CH=2AH$. But the altitudes of triangles AGH and CGH are the same.

$$\therefore \triangle AGH = \frac{1}{2} \triangle CGH.$$

For the same reason as above, $BH=2GH$.

$$\therefore BG=3GH.$$

The altitudes of triangles AGH and ABG being the same, $\triangle AGH = \frac{1}{3} \triangle ABG$. Since the homologous sides in triangles AGH and BCH are one-half the size in the former of those in the latter, $\triangle AGH = \frac{1}{4} \triangle BCH$.

Also solved by J. Scheffer, Francis Rust, M. A. Muzzy, and H. C. Feemster.

A. M. Harding should have received credit for solving 397.

399. Proposed by J. K. ELLWOOD, Superintendent of Schools, Lucas, Kansas.

A race track is to be composed of two tangents and the arc of the circle which is concave towards the point of intersection of the two tangents, each tangent and the arc of the circle being 1 mile. What is the radius of the circle?

Solution by CHRISTIAN HORNUNG, Heidelberg University, Tiffin, Ohio.

Let AT and BT be the tangents, and ACB the arc composing the track; O the center, and OA the radius of the circle.

Let $AOT = \theta$, and $AO = r$. Then arc $ACB = (2\pi - 2\theta)r = 1$, and $\tan \theta = 1/r$; whence $\tan \theta = 2(\pi - \theta)$.

This equation solved by approximation gives

$$\theta = 74^\circ 46'.18. \quad r = \frac{1 \text{ mile}}{\tan \theta} = \frac{5280 \text{ feet}}{\tan \theta} = 1437.45 \text{ feet.}$$

Also solved by J. E. Sanders, A. M. Harding, J. Scheffer, H. Prime, A. H. Holmes, Elmer Schuyler, E. B. Escott, and H. C. Feemster.

CALCULUS.

321. Proposed by ARTEMAS MARTIN, Ph. D., LL. D., United States Coast and Geodetic Survey Office, Washington, D. C.

To a person in a boat at the center of a circular pond the bottom appears to be perfectly level. What is the actual form of the bottom of the pond, the depth of the water at the center being a feet, and the distance of the eyes of the observer from the surface of the water being b feet. [From the *Mathematical Visitor*, Vol. 2, No. 2, p. 62.]

Solution by E. B. ESCOTT, Ann Arbor, Michigan.

Let A be the bottom of the pond at the center, OX the surface of the water, B the observer. Then $\sin \theta = m \sin \phi$.

Let $OQ=q$, $\sin \theta = \frac{q}{\sqrt{q^2+b^2}}$, $\sin \phi = \frac{q}{m \sqrt{q^2+b^2}}$;

$$QP=QR=\frac{a}{b}. BQ=\frac{a}{b}\sqrt{q^2+b^2}; x=q+\frac{aq}{bm}; y=QP\cos\phi$$

$$=a\sqrt{1+\frac{(m^2-1)x^2}{(a+bm)^2}}; \text{ i. e., } \frac{y^2}{a^2}-\frac{x^2}{(a+bm)^2}=\frac{m^2-1}{m^2-1}=1,$$

hyperbola, center at O , AB principal axis.

An excellent solution was also received from H. C. Feenster.
This problem properly belongs to the Miscellaneous Department.

322. Proposed by E. B. ESCOTT, University of Michigan.

Find the equation of the curve such that the solid of revolution generated by revolving it about the x -axis shall have a volume equal to the m/n th part of the volume of the circumscribed cylinder.

Solution by H. PRIME, Boston, Massachusetts, and J. SCHEFFER, A. M., Hagerstown, Maryland.

$$\text{Let } \pi \int y^2 dx = \pi (m/n) y^2 x.$$

Differentiating and arranging, $2m dy/y = (n-m) dx/x$.

Hence by integrating, $2m \log y = (n-m) \log x + \log c$.

$$\log y^{2m} = \log x^{n-m} + \log c, \quad y^{2m} = a^m x^{n-m} \text{ and } y^2 = ax^{n/m-1}.$$

This is the equation of the family of parabolas, including the case when $m/n=1/3$ and $y^2=ax^2$. Thus we have the familiar relation of the cone and its circumscribing cylinder.

Also solved similarly by M. A. Muzzy, and Elijah Swift.

323. Proposed by C. N. SCHMALL, New York City.

From what height must an elastic ball be dropped in order that, after impact with the hard surface of the sidewalk, it may rebound to a given altitude a in the least possible time from the moment of descent?

Solution by J. SCHEFFER, A. M., Hagerstown, Maryland.

Let x be the height. Then will the velocity at the moment the ball strikes the surface $=\sqrt{2gx}$, and the time, $t_1, =\sqrt{2x/g}$. It rebounds now with a velocity $=e\sqrt{2gx}$, e being the coefficient of elasticity. Denoting the time it reaches the height a by t_2 , we have $a=e\sqrt{2gx}.t_2-(gt_2^2/2)$, whence $t_2=\sqrt{2/g}[e\sqrt{x}\pm\sqrt{e^2x-a}]$. Consequently the total time is